INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS
Int. J. Numer. Meth. Fluids 2002; **38**:555–566 (DOI: 10.1002/fld.231)

Three dimensional tide-induced circulation model on a triangular mesh

Wensheng Jiang*,† and Wenxin Sun

Institute of Physical Oceanography, Ocean University of Qingdao Yushan Road 5, 266003 Qingdao, People's Republic of China

SUMMARY

In this paper a triangular mesh, which is generated by an automatic triangular mesh generator, is used to calculate the tide-induced residual current in the Bohai Sea, China. The generator can modify the distribution of the node according to the water depth, thus ensuring the uniform distribution of the Courant number. The three dimensional nonlinear tide equation is employed to get the Eularian residual current and Stokes' drift. Then the Lagrangian residual current is obtained by adding the above two values. It has been proved in other papers that the Lagrangian residual current is more proper to represent the circulation and a numerical example is given. The application of these methods to the Bohai Sea shows that the results are in accordance with those obtained before. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: triangular mesh; numerical modelling; circulation; Bohai Sea

1. INTRODUCTION

The numerical simulation method is widely used in physical oceanography. Normally the partial differential equation is solved numerically on a rectangular mesh and then the results are obtained. Generally this turns out to be rather successful in many cases but it is not always an appropriate solution to problems encountered. For example, if the boundary of the studied area is complicated, the rectangular mesh cannot represent the boundary very well. Another example is that sometimes in some areas the space resolution needs to be high, yet normally the regular mesh is not flexible enough to fulfil this requirement. Based on the above discussion, the irregular mesh is introduced in the numerical simulation of physical oceanography [1–3].

In physical oceanography the circulation is one of the basic problems. In shallow water areas tidal movement is dominant, so the tide-induced residual currents are essential in the

*Correspondence to: Wensheng Jiang, Institute of Physical Oceanography, Ocean University of Qingdao, Yushan Road 5, Qingdao 266003, People's Republic of China.
†E-mail: wsjang@ouqd.edu.cn

Received November 1999
Revised 14 June 2001

circulation. Feng and Cheng [4, 5] pointed out that the Lagrangian residual currents are the proper representation of the tide-induced circulation. In this paper a three dimensional (3D) numerical scheme with σ coordinate in the vertical direction is developed on a triangular mesh to calculate the tide-induced current in the Bohai Sea in China.

2. AUTOMATIC TRIANGULAR MESH GENERATION

The triangular mesh used in this paper is generated automatically on the basis of a method developed by Kashiyama and Okada [3]. Detailed description of the method will not be discussed here. Only the basic concept is mentioned.

The Courant number can be defined as

$$C = U\Delta t/\Delta x$$

where C is the Courant number, U is the velocity, Δt is the time interval in the integration and Δx is the element size. Assuming the long-wave theory the velocity U can be expressed as

$$U = \sqrt{gh}$$

Then the Courant number is

$$C = \sqrt{gh}\Delta t/\Delta x$$

In the numerical simulation the Courant number is a restricting factor to numerical stability. The key feature of this method is that it can generate the mesh with the same Courant number, which, in turn, can improve the numerical stability and the accuracy. In a linear wave system the Courant number is the criteria of stability. Although in a nonlinear system no such criteria is defined the Courant number can still be regarded as a somewhat necessary condition for stability. Thus it is important to have a similar Courant number. According to the expression of the Courant number the structure of the triangular mesh is controlled by the topography of the sea area.

3. TIDAL CURRENT MODEL

3.1. Governing equations for tidal movement

The three dimensional barotropic equations for tidal movement can be expressed as

$$\begin{cases} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = 0 \\ \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + w\dfrac{\partial u}{\partial z} - fv = -g\dfrac{\partial \zeta}{\partial x} + \dfrac{\partial}{\partial z}\left(\gamma\dfrac{\partial u}{\partial z}\right) \\ \dfrac{\partial v}{\partial t} + u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + w\dfrac{\partial v}{\partial z} + fu = -g\dfrac{\partial \zeta}{\partial y} + \dfrac{\partial}{\partial z}\left(\gamma\dfrac{\partial v}{\partial z}\right) \end{cases} \tag{1}$$

when $z=\zeta,\ w=\dfrac{\partial\zeta}{\partial t}+u\dfrac{\partial\zeta}{\partial x}+v\dfrac{\partial\zeta}{\partial y}$

$$\left(\gamma\frac{\partial u}{\partial z},\gamma\frac{\partial v}{\partial z}\right)=0$$

$$z=-h,\quad w=-u_b\frac{\partial h}{\partial x}-v_b\frac{\partial h}{\partial y}$$

$$\left(\gamma\,\frac{\partial u}{\partial z},\gamma\,\frac{\partial v}{\partial z}\right)=C_{\mathrm{D}}\sqrt{u_b^2+v_b^2}(u_b,v_b)$$

where u,v,w are velocities in (x,y,z) directions, ζ is the water elevation, f is the Coriolis parameter and g is the acceleration of gravity. γ is the eddy viscosity parameter, (u_b,v_b) is the horizontal velocity at sea bottom, h is water depth and C_{D} is drag coefficient at sea bottom. It is obvious to see that it is a nonlinear system, because the advection term in the momentum equation and the boundary condition at the sea surface show the nonlinear interaction between the current velocity and water elevation.

3.2. Numerical scheme

The common σ transformation in the vertical direction is applied to Equation (1). The momentum equations in Equation (1) are solved implicitly in the vertical direction, that is, the time differential term and the vertical eddy viscosity term compose a vertical system and all the other terms are treated explicitly. Thus a tri-diagonal matrix is formed which can be solved easily.

The vertical velocity w^* and water elevation ζ are solved by integrating the continuity equation. After discretization the numerical scheme for water elevation can be expressed as

$$\zeta_i^{n+1}=\zeta_i^n-\Delta t\left(\frac{\partial}{\partial x}\left(H\int_0^1 u_i^n\mathrm{d}\sigma\right)+\frac{\partial}{\partial y}\left(H\int_0^1 v_i^n\mathrm{d}\sigma\right)\right) \tag{2}$$

Here the subscript i means the different node in the horizontal direction.

As for vertical velocity w^* the continuity equation is integrated from the sea bottom to the jth layer, then w_j^* can be acquired.

$$\frac{\partial}{\partial x}\left(H\int_0^{\sigma_j}u\mathrm{d}\sigma\right)+\frac{\partial}{\partial y}\left(H\int_0^{\sigma_j}v\mathrm{d}\sigma\right)+Hw_j^*+\sigma_j\frac{\partial\zeta}{\partial t}=0$$

where σ_j is the σ value of the jth layer. Now w^* is obtained.

In the horizontal direction all the variables such as u, v, and ζ are on the same node of the triangular mesh. The horizontal derivatives of u, v and ζ at one point are calculated according to the values of its surrounding nodes. The structure is shown in Figure 1. The formation of the horizontal derivatives are based on Thacker [1] which is given below.

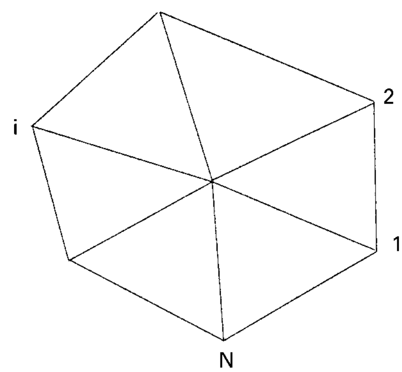

Figure 1. The node structure for derivatives.

Let f be a function its derivatives can be expressed as follows on a triangular mesh.

$$\begin{aligned}\frac{\partial f}{\partial x} &= \sum_{i=1}^{N} f_i(y_{i+1} - y_{i-1}) \Big/ \sum_{i=1}^{N} x_i(y_{i+1} - y_{i-1}) \\ \frac{\partial f}{\partial y} &= \sum_{i=1}^{N} f_i(x_{i+1} - x_{i-1}) \Big/ \sum_{i=1}^{N} x_i(y_{i+1} - y_{i-1})\end{aligned} \tag{3}$$

The points i from 1 to N are anticlockwise and shown in Figure 1. If $i-1$ is less than zero it should be N and if $i+1$ is greater than N it is 1.

4. LAGRANGIAN RESIDUAL CURRENT MODEL

In the shallow sea the oscillating tidal current is one of the dominant movements. Because it is a nonlinear system the trajectory of one particle in a tidal period will not be enclosed. Thus the Lagrangian mean velocity is formulated as

$$\vec{u}_{\mathrm{L}} = \frac{1}{nT} \int_{t_0}^{t_0+nT} \vec{u}(\vec{x}(\vec{x}_0, t), t)\, \mathrm{d}t \tag{4}$$

where $\vec{u}_{\mathrm{L}}$ the Lagrangian mean velocity, $\vec{u}$ is the tidal current, T is the period of the tide, $\vec{x}_0$ is the starting point of one particle and $\vec{x}$ is the position of the particle.

Another way to understand the tide-induced residual current is to eliminate the periodical part of the tidal currents with harmonic analysis at one specific point. The result is called Eulerian residual current, which is defined as

$$\vec{u}_{\mathrm{E}} = \frac{1}{nT} \int_{t_0}^{t_0+nT} \vec{u}(\vec{x}_0, t)\, \mathrm{d}t \tag{5}$$

where $\vec{u}_{\mathrm{E}}$ is Eulerian residual current and the notation for the other symbols remains the same as for Equation (1).

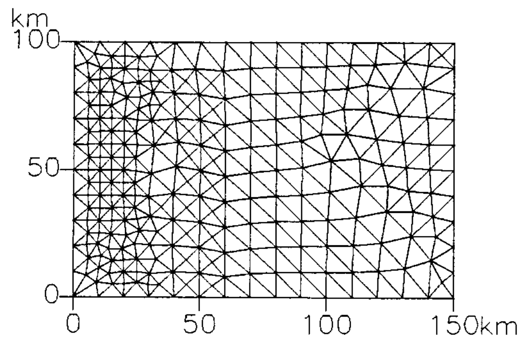

Figure 2. The structure of the model area.

The relationship between the Lagrangian residual current and the Eulerian residual current is outlined by Longuet-Higgins [6] and listed below.

$$\vec{u}_{\mathrm{L}} = \vec{u}_{\mathrm{E}} + \vec{u}_{\mathrm{S}} \tag{6}$$

where $\vec{u}_{\mathrm{S}} = \langle \int_{t_0}^{t} \vec{u}(\vec{x}_0, t)\,\mathrm{d}t \cdot (\nabla \vec{u}(\vec{x}_0, t)) \rangle$ is the Stokes' drift, while $\langle\ \rangle$ is an operator which is expressed as $\langle \cdot \rangle = (1/T) \int_{t_0}^{t_0+T} \cdot\, \mathrm{d}t$.

Hydrodynamic equations for the Eulerian residual current [7, 8] and the corresponding mass transport equations [9] were deduced. Nevertheless, there are false sink and source term in these equations. Feng and Cheng [4, 5] promoted a weak nonlinear theory for the shallow sea circulation and suggested using the Lagrangian residual current instead of the Eulerian current to describe the long-term mass transport. This avoids the weak point in Eulerian residual currents.

In order to obtain the Lagrangian residual current a tidal movement should be solved first. Then Eulerian residual current and the Stokes' drift can be calculated easily. According to Equation (6) the Lagrangian residual current can be deduced.

5. NUMERICAL RESULTS

5.1. Test run in a rectangular domain

In order to test the performance of the numerical scheme described above a rectangular domain is used to serve as an example. The length of the sea area is 150 km and the width is 100 km. The depth of the sea area is from 5 m at the west to 15 m at the boundary which is the open boundary. The structure of the grids is given in Figure 2. At the open boundary an M2 tide with the amplitude of 1.1 m is imposed.

The surface Eulerian residual current is shown in Figure 3. It is clear to see an anti-clockwise circulation in the model area. At the open boundary the water moves out of the area. The pattern is also the same for the other layers of the water. This evidently violates the conservation of mass. In Figure 4 the surface Stokes' drift is displayed. Contrary to the Eulerian residual current the Stokes' drift points to the inside of the model area at the open boundary. The Lagrangian residual current is obtained in Figure 5 by adding up these two.

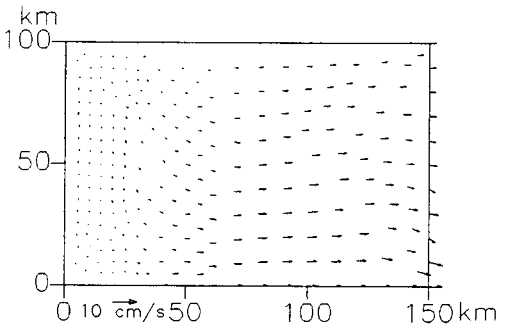

Figure 3. The Eularian residual current of the model area.

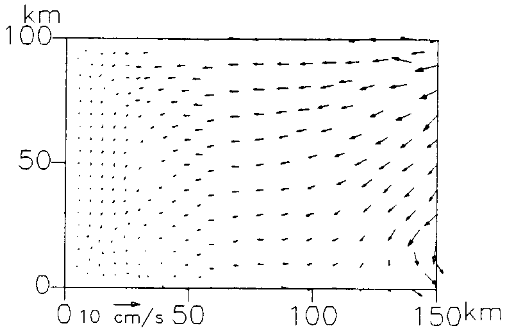

Figure 4. The stokes' drift of the model area.

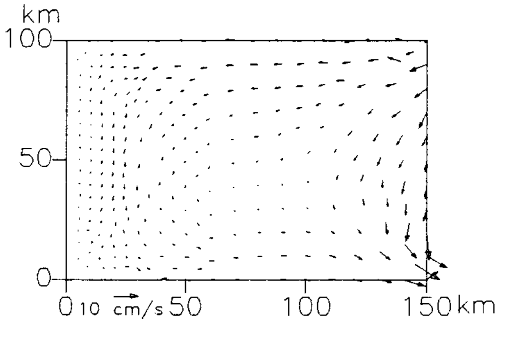

Figure 5. The Lagrangian residual currents of the model area.

It is obvious there is an anti-clockwise circulation and the conservation law is fulfilled. At least in this aspect the Lagrangian residual current is more appropriate in representing the circulation in the coastal sea.

5.2. Application to the Bohai sea

The Bohai Sea is a semi-enclosed continental shelf sea located between 37°07′N to 41°N and 117°35′E to 121°10′E. It is surrounded by land in the north, west and south and only connected to the Yellow Sea through the narrow Bohai Strait. The Bohai Sea extends to 300 km from west to east and 550 km from south to north with a total area of 80 000 km^2. It is divided into four parts: the Liaodong Bay in the northeast, the Bohai Bay in the west, the Laizhou Bay in the south and the central basin. The Bohai Sea is very shallow with depths less than 30 m in 95 per cent of the area. The average water depth is only 18 m. The deepest part is the Laotieshan water way with up to 86 m in depth (see Figure 6). In the Bohai Sea the tidal movement is one of the prominent hydrodynamic processes. The tidal wave from the Yellow Sea propagates through the Bohai Strait into the Bohai Sea of which the M2 tide is the most important one.

Figure 7 is the grid distribution generated by the algorithm mentioned above. There are 1915 nodes together which forms 3584 triangular meshes. There are 367 (10.53 per cent of the total) meshes with an angle greater than 90 degrees and only 32 (0.92 per cent of the total) with an angle greater than 120 degrees. The maximum of the angle is 157. In this arrangement the longest length of the triangular mesh is around 20 km and the shortest is only 0.8 km. It is clear to see the correlation between the distribution of the nodes and the topography. For example the nodes are coarse in the central part of the Bohai Sea and the northern part of the Bohai Strait.

In the model the M2 tide is imposed at the open boundary. Figure 8 is the co-amplitude and co-phase map of the M2 tide calculated by this model. It shows good accordance with the one observed in Marine Atlas of Bohai Sea, Yellow Sea and East China Sea [10]. Figures 9 and 10 are the calculated and observed surface elliptical axis of tidal current for M2 component respectively. The similarity between these two figures can easily be seen. All of the above shows that the present model reproduces the M2 tide in the Bohai Sea correctly.

The Lagrangian residual currents are shown in Figure 11. Generally the Lagrangian current is less than 10 cm s^{-1} and decreases with depth. In the central region the velocity is less than 1 cm s^{-1}. In Figure 11 two anti-clockwise circulation patterns can be found although they are not very clear especially in the Liaodong Bay. Sun *et al*. [11] calculated the tide-induced Lagrangian residual currents in the Bohai Sea in a rectangular grid. The results also show two anti-clockwise current patterns in the Liaodong Bay and the Bohai Bay and are in accordance with the others [12, 13].

Another feature is that there is a current along the southern coast of the Laizhou Bay and towards the outside of the Bohai Sea. This is supported by the study of suspended particulate transport by Jiang and Mayer [14]. According to their study the SPM from the Yellow River goes out of the Bohai Sea along the southern coast of the Laizhou Bay.

From Figure 11 it can be seen that the water flows in the Bohai Sea through the north of the Bohai Strait and flows out through the south of the strait. This has been observed during the investigation and can be found in Figure 11. Since the wind plays an important role in the circulation pattern the feature of the results here is

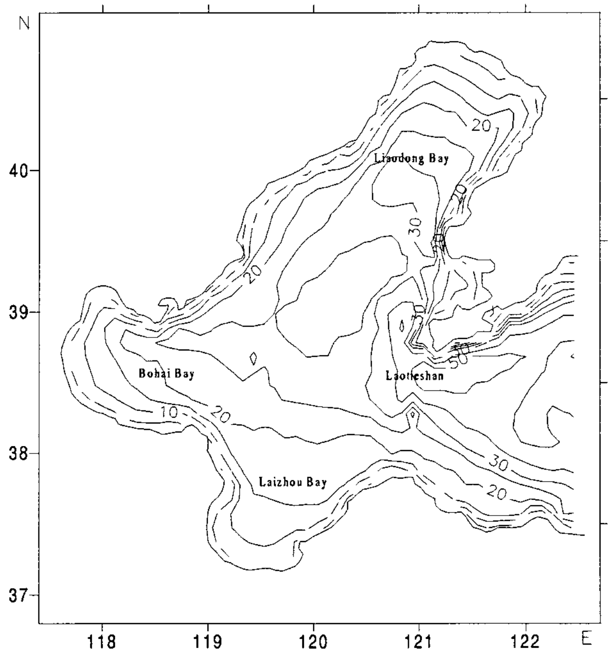

Figure 6. The topography of the Bohai Sea.

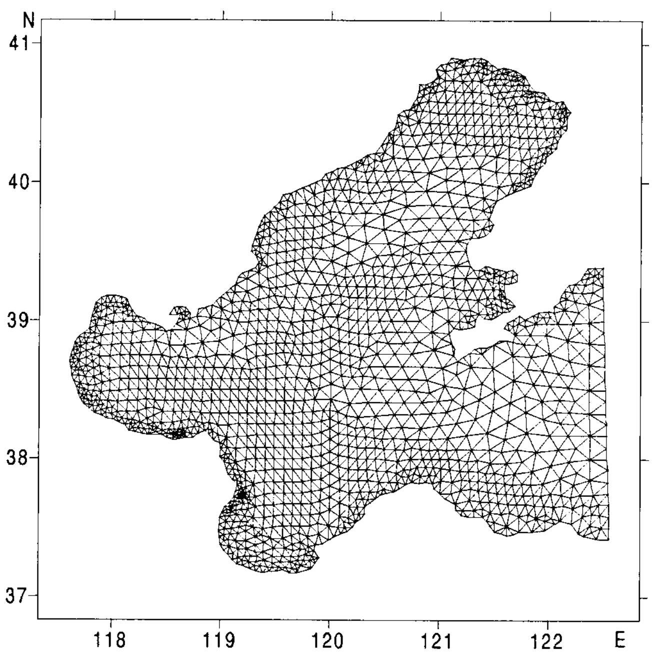

Figure 7. The grid arrangement of the Bohai Sea.

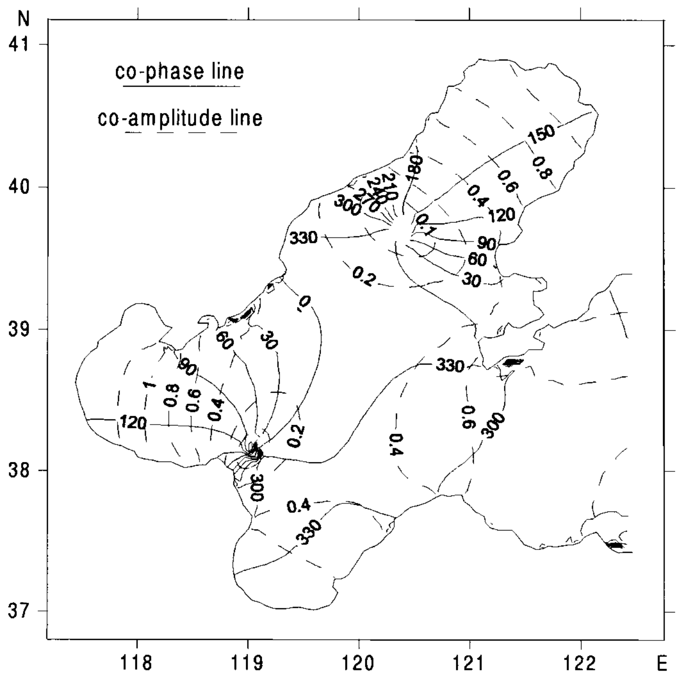

Figure 8. The calculated M2 co-amplitude and co-phase map of the Bohai Sea.

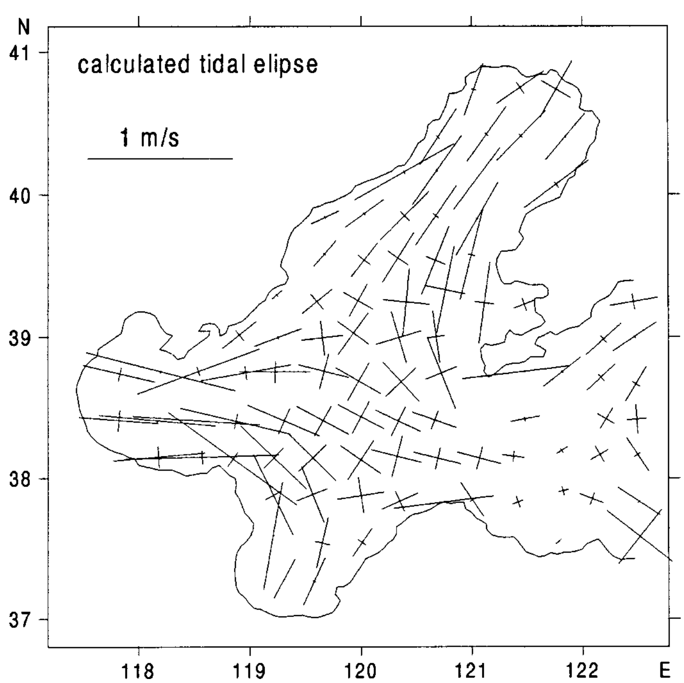

Figure 9. The calculated surface elliptical axis of M2 tidal current in the Bohai Sea.

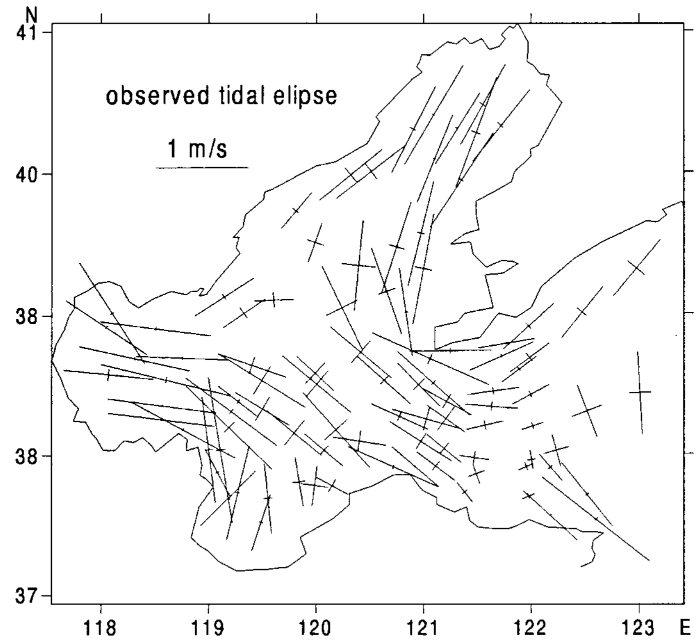

Figure 10. The observed surface elliptical axis of M2 tidal current in the Bohai Sea (in *Marine Atlas of Bohai Sea*, *Yellow Sea and East China Sea* [10]).

not very clear. But the tide-induced residual current cannot be neglected in the shallow sea area.

6. CONCLUSIONS

The triangular mesh is used in the study of circulation in the shallow sea. The results are compatible with those calculated on the rectangular mesh. The flexibility of the triangular mesh facilitates the simulation of the shallow sea process. It can conform to the complex boundary and change its size according to the topography. This can be used in the modelling in estuarine and other small complex geographical systems.

Another potential application of the triangular mesh to the marine system modelling is that it can increase the resolution in the interested area. This can be used to catch the front where the large gradient of the physical parameter appears.

In this paper a numerical example reveals that it is more appropriate to use the Lagrangian residual current than the Eulerian residual current in representing the circulation. This has been proved before. The application of the model to the Bohai Sea shows that the results are as good as those calculated on the rectangular grid in general, but give more detailed information near the boundary area.

There is, however, a disadvantage of applying the triangular mesh from the perspective of computational mathematics. The irregular mesh can lead to instability and the pseudo-reflection and pseudo-refraction may occur. For example in this case the time step is less

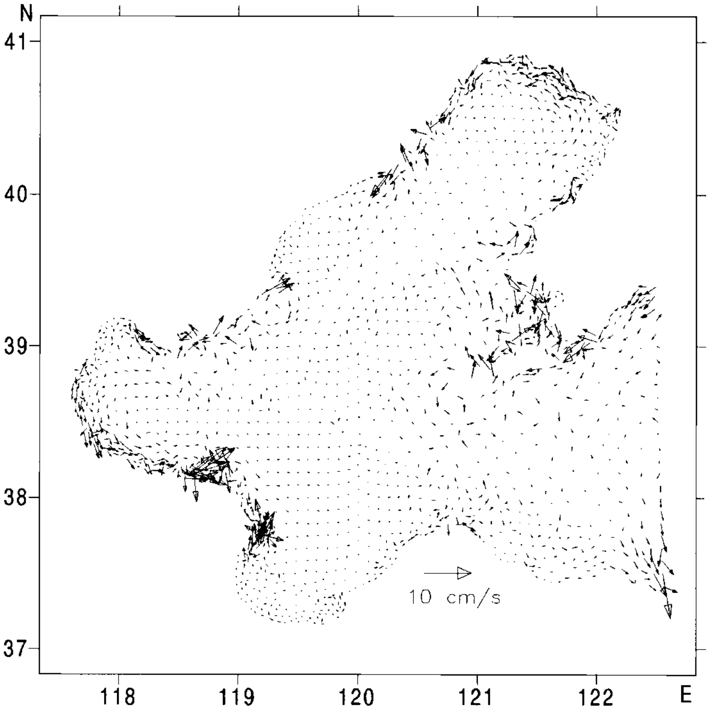

Figure 11. The M2 tide-induced Lagrangian surface residual current in the Bohai Sea.

than 15 s otherwise it is unstable. As for the pseudo-reflection and pseudo-refraction it can be seen from Figure 8 that the co-amplitude and co-phase line are not smooth which shows the contamination by the noise. The main idea to overcome this effect is to distribute the nodes uniformly, yet this problem is far from solved theoretically and needs to be studied further.

ACKNOWLEDGEMENTS

This work is supported by National Natural Science Foundation of China with grant number 49806007 and by the major state basic research program, No. G1999043804.

REFERENCES

1. Thacker WC. Irregular-grid finite-difference techniques for storm surge calculations for curving coastlines. In Marine forecasting, prediction and modelling in ocean hydrodynamics. *Proceedings of the 10th Liege Colloquium on Ocean Hydrodynamics*, Elsevier, 1979; 261–283.
2. Shi FY, Sun WX. A variable boundary model of storm surge flooding in generalized curvilinear grids. *International Journal for Numerical Methods in Fluids* 1995; **21**(8):641–651.
3. Kashiyama K, Okada T. Automatic mesh generation method for shallow water flow analysis. *International Journal for Numerical Methods in Fluids* 1992; **15**:1037–1057.
4. Feng SZ, Cheng RT, Xi PG. On tide-induced Lagrangian residual current and residual transport, Part I: Residual Current. *Water Resource Research* 1986; **22**(12):1623–1634.

5. Feng SZ, Cheng RT, Xi PG. On tide-induced Lagrangian residual current and residual transport, Part II: Residual transport with application in South San Francisco Bay, California. *Water Resource Research* 1986; **22**(12): 1635–1646.
6. Longuet-Higgins MS. On the transport of mass by time-varying currents. *Deep-Sea Research* 1969; **16**:431–447.
7. Nihoul JCJ, Ronday FC. The influence of the 'tidal stress' on the residual circulation. *Tellus* 1975; **27**:484.
8. Heaps NS. Linearized vertically-integrated equations for residual circulation in coastal seas. *Deutsche Hydrografische Zeitschrift* 1978; **31**:147.
9. Dyer KR. *Estuaries: A Physical Introduction*. Wiley: New York, 1973.
10. *Marine Atlas of Bohai Sea*, *Yellow Sea and East China Sea*, *1992*. China Ocean Press: Beijing.
11. Sun WX, Xi PG, Song LN. Numerical calculation of the three dimensional tide-induced Lagrangian residual circulation in the Bohai Sea. *Journal of the Ocean University of Qingdao* 1989; **19**(2):135–144.
12. Le KT, Feng M, Wang Y. A numerical study of the wintertime circulation in the Bohai and Huanghai Seas. *Chinese Journal of Oceanography and Limnology* 1993; **11**(2):149–160.
13. Miao JB, Liu XQ. Numerical experiments on dynamics of the wintertime circulation in the northern Huanghai and Bohai Seas. *Acta Oceanologica Sinica* 1989; **11**(1):15–22 (in Chinese).
14. Jiang WS, Mayer B. A study on the transport of suspended particulate matter from the Yellow River by using a 3D particle model. *Journal of the Ocean University of Qingdao* 1997; **27**(4):438–445 (in English with Chinese abstract).

